Results and Frontiers in Lattice Baryon Spectroscopy

John Bulava[1]*, Robert Edwards†, George Fleming**, K. Jimmy Juge‡, Adam C. Lichtl§, Nilmani Mathur¶, Colin Morningstar*, David Richards† and Stephen J. Wallace‖

*Department of Physics, Carnegie Mellon University, Pittsburgh, PA 15213, USA
†Thomas Jefferson National Accelerator Facility, Newport News, VA 23606, USA
**Yale University, New Haven, CT 06520, USA
‡Department of Physics, University of the Pacific, Stockton, CA 95211, USA
§RBRC, Brookhaven National Laboratory, Upton, NY 11973, USA
¶Tata Institute of Fundamental Research, Mumbai 40005, India
‖University of Maryland, College Park, MD 20742, USA

Abstract. The Lattice Hadron Physics Collaboration (LHPC) baryon spectroscopy effort is reviewed. To date the LHPC has performed exploratory Lattice QCD calculations of the low-lying spectrum of Nucleon and Delta baryons. These calculations demonstrate the effectiveness of our method by obtaining the masses of an unprecedented number of excited states with definite quantum numbers. Future work of the project is outlined.

A main goal of the LHPC is to determine the spectrum of excited baryons. This is achieved by extracting hadron masses from the exponential fall-off of two-point temporal correlation functions $\langle 0|\mathscr{O}_i(t)\bar{\mathscr{O}}_j(0)|0\rangle$, in which the operators $\mathscr{O}_i$ are composed of quark and gluon fields. These correlators can be expressed in terms of path integrals on a discrete space-time lattice [1] which are calculated numerically using Monte Carlo techniques [2].

The method developed by the LHPC to extract baryonic spectra is detailed in Refs. [3, 4, 5]. This involves the construction of interpolating operators to create states with definite quantum numbers, the evaluation of the temporal correlators $\langle 0|\mathscr{O}_i(t)\bar{\mathscr{O}}_j(0)|0\rangle$, and the pruning of the final operator sets.

If we have a set of N operators $\{\mathscr{O}_i\}$, we may form the $N\times N$ (time-dependent) matrix

$$C_{ij}(t) = \langle 0|\mathscr{O}_i(t)\bar{\mathscr{O}}_j(0)|0\rangle. \tag{1}$$

The diagonalization of $C^{-1/2}(t_0)C(t)C^{-1/2}(t_0)$, where t_0 is a small non-zero reference time, produces correlators that may be fit to decaying exponentials to obtain E_i, the energies of the lowest-lying states that can be interpolated by $\{\mathscr{O}_i\}$. If only zero momentum operators are used, the extracted energies of single particle states correspond to their masses.

---

[1] Speaker, jbulava@andrew.cmu.edu

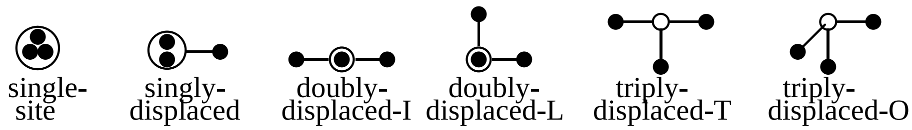

FIGURE 1. The various types of extended baryon operators. Solid circles represent smeared quark fields, lines represent smeared link fields, and hollow circles the location of the reference site.

Our baryon operators are composed of covariantly displaced quark fields as building blocks combined in a gauge invariant way to interpolate both ground and excited baryonic states. Both the quark and link fields are *smeared* (replaced with a spatially localized weighted average) [4] to increase overlap with low-lying modes. Furthermore, they are designed to be *spatially extended* in order to interpolate states which are radially and orbitally excited. This spatial extension is achieved by gauge-covariantly displacing some (or all) of the three quarks in different directions from some reference site. The various types of extended baryon operators are shown in Fig. 1.

Group theoretical projections are used to construct operators which transform irreducibly under the symmetries of the lattice. Recall that a baryonic state is denoted by its mass (m), spin and parity (J^P), and isospin (I). To construct states with definite isospin, we combine quark fields of different flavors to create an operator that transforms irreducibly under SU(2) isospin. Since the QCD Lagrangian is diagonal in flavor (and thus isospin) space, members of an iso-multiplet will be mass degenerate. Thus we may arbitrarily choose the operator with maximal third component of isospin I_3 as the interpolating operator for a given iso-multiplet.

In continuous space, J^P labels an irreducible representation of the (double-valued) rotation group SU(2), which is a symmetry group of rotationally invariant Hamiltonians. However, on a cubic lattice the symmetry group is no longer that of continuous rotations but the subgroup of discrete lattice rotations. This group is denoted O_h^D and is termed the double cubic point group. Because this group has a finite number of elements, it also has a finite number of irreducible representations, in contrast to the infinite number of possible J^P values in continuous space. These representations are labeled $\mathrm{G_{1g}, H_g, G_{2g}, G_{1u}, H_u,}$ and $\mathrm{G_{2u}}$ for historical reasons. There also exists a procedure known as *subduction* to identify the states appearing in these irreducible representations with continuum J^P values. For a complete review of the operator construction procedure, see Ref. [3].

After employing these considerations, we have operators which interpolate states with definite isospin, lattice spin, and parity. However, for a particular isospin (e.g. the Nucleons or Delta baryons) and lattice spin-parity value (such as $\mathrm{G_{1g}}$ or $\mathrm{H_g}$), this operator set is unmanageably large. It is necessary to optimize or ‘prune’ this set down to a manageable number [5].

The operator set is first pruned based on intrinsic noise. This amounts to selecting the ten least noisy operators within each operator type (single-site, singly-displaced, *etc.*). These operators are determined by examining the diagonal correlators $\langle 0|\mathscr{O}_i(t)\overline{\mathscr{O}}_i(0)|0\rangle$. We find that these least noisy operators have good overlap with the low-lying states of interest and couple weakly to the higher-lying energy modes of the theory.

In addition, noise enters if the operator set produces a correlation matrix which is ill-conditioned. After the least noisy set of operators of each type is determined, the subset of those operators with the lowest condition number is chosen. This condition number is calculated for the normalized correlation matrix $C_{ij}(t')/\sqrt{C_{ii}(t')C_{jj}(t')}$, where $C_{ij}(t)$ is defined in Eq. 1 and evaluated at some early time $t' \approx 3a_t$. In determining the optimal subset of operators, both condition number and operator type diversity are taken into consideration.

An exploratory calculation designed to test the effectiveness of the method has been performed for the Nucleon [5] and Delta baryons. To this end, calculations were carried out on a small volume ($\approx$ 1.2fm) lattice with a relatively coarse ($\approx$ 0.1fm) lattice spacing. For computational simplicity, the pion mass for this calculation was unphysically large ($m_\pi \approx$ 700MeV) and the quenched approximation (omission of quark loops) was employed. Also, these results were based on a low-statistics ensemble of 200 configurations.

In Fig. 2, the spectrum for each lattice J^P value ($\mathrm{G_{1g}}, \mathrm{H_g}$, etc.) is plotted in lattice units. Comparison is made with experimental results [6] by identifying these lattice spin-parity values with their continuum counterparts using the group theoretical process of subduction. It should be noted that for $J \geq \frac{5}{2}$, one continuum state will have several lattice counterparts. The scales have been set using the mass of the lowest-lying hadron, the $P_{11}(939)$. Shown here are the experimental and lattice results for the Deltas. For an analogous plot of the Nucleons, see Ref. [7].

Upon examination of Fig. 2, we notice that the lowest-lying energy state corresponds to a $J^P = \frac{3}{2}^+$ ($\mathrm{H_g}$) state, in agreement with the experimental $P_{33}(1232)$. The next two states occur in the $\frac{1}{2}^-$ ($\mathrm{G_{1u}}$) and $\frac{3}{2}^-$ ($\mathrm{H_u}$) channels, in agreement with the experimental results. However, the first even parity excitation, the $P_{33}(1600)$, disagrees with the experimental result. This feature is also present in the nucleon sector [7], where the $P_{11}(1440)$ Roper resonance is higher than the first odd-parity excitation. As our simulation neglected quark loops and possessed an unphysically high pion mass, this discrepancy indicates that either these states are not interpolated by standard 3-quark operators (in the quenched approximation) or that they are sensitive to quark mass effects.

The exploratory calculation described here will be followed by unquenched high-statistics runs at larger volumes, finer lattice spacings, and lower quark masses. Computational time has already been allotted for such runs using 10 million core-hours on the Cray XT3 at Oak Ridge National Lab and 3 million service units on the Cray XT3 'bigben' at the Pittsburgh Computing Center.

Future work will utilize 'all-to-all' propagators [8], which estimate quark propagators from all initial sites to all final sites. This is in contrast with the currently used 'point-to-all' propagators which only employ one initial site. This method will result in an increase in statistics, but more importantly enable the construction of multi-hadron operators.

In conclusion, this exploratory calculation of the low-lying Nucleon and Delta spectra by the LHPC demonstrates the extraction of an unprecedented number of excited states in Lattice QCD. With the allocation of future computer resources and the development of all-to-all propagators, our collaboration continues to move toward the ultimate goal of identifying states with definite quantum numbers, predicting their masses, and calculating matrix elements.

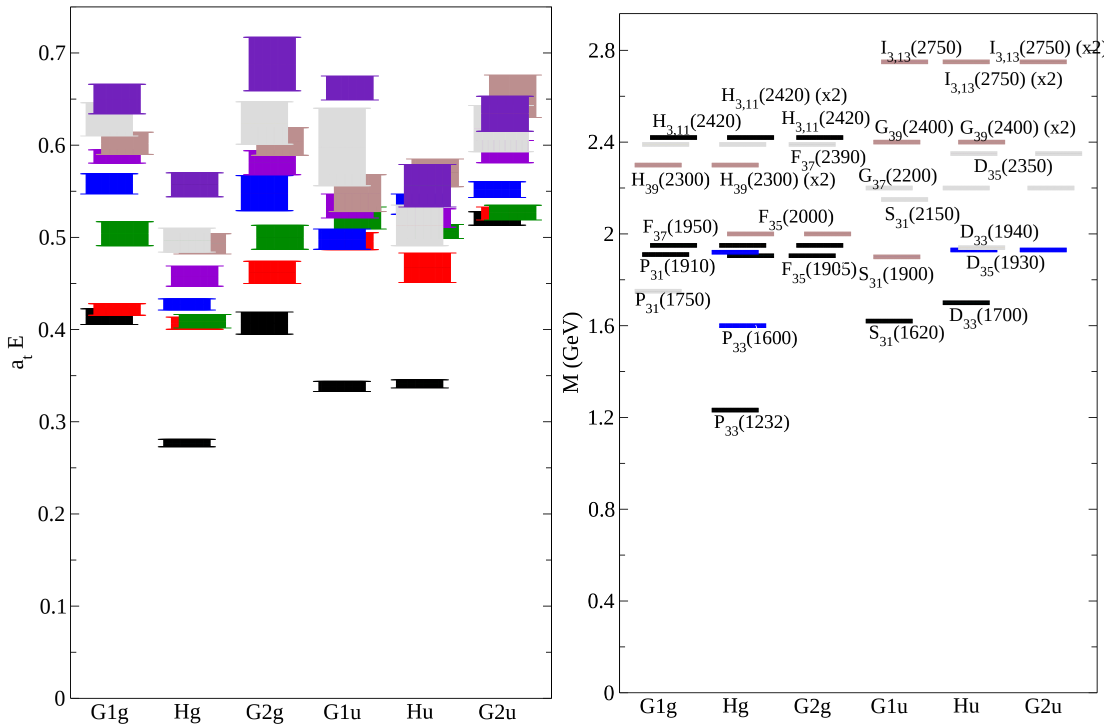

FIGURE 2. The low-lying Delta spectrum. The left plot is the lattice calculation (colors are used to distinguish different levels). In the experimental plot (right), black denotes a 4-star state, blue a 3-star state, brown a 2-star state, and gray a 1-star state. States are grouped according to their lattice spin-parity values, G_{1g}, H_g, etc.

This research is supported by NSF grant PHY 0653315 and Dept. of Energy contracts DE-AC05-06OR23177 and DE-FG02-93-ER-40762. Travel to the VII Latin American Symposium on Nuclear Physics and Applications was made possible by the NSF Sponsorship Program.

REFERENCES

1. I. Montvay and G. Munster, *Quantum Fields on a Lattice*, Cambridge University Press, 1994; H.J. Rothe, *Lattice Gauge Theories: An Introduction*, 2nd Ed., World Scientific Publishing Co., 1997.
2. C. Morningstar, hep-lat/0702020.
3. S. Basak et al., Phys. Rev. **D72** (2005), 094506.
4. A. C. Lichtl et al., PoS **LAT2005** (2006), 076.
5. A. C. Lichtl, *PhD Thesis*, hep-lat/0609019
6. W.M. Yao et al., *Review of Particle Physics*, J. Phys. **G33** (2006), 1-1232.
7. A. C. Lichtl, *These Proceedings*.
8. J. Foley et al., Comput. Phys. Commun. **172** (2005), 145.